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Inherent Conservatism in Deterministic Quasi-Static Structural Analysis

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NOMENCLATURE

C	= Constant coefficient
K	= Sample size tolerance factor
L	= Loads, kips
M	= Moment, kip-in
N	= Probabilistic range factor
n	= Statistical sample number
P	= Probability
S	= Stress, ksi
R	= Reliability
SF	= Conventional safety factor
W	= Weight, kips
x	= Specimen value
Z	= Safety index
η	= Coefficient of variation, σ/μ
μ	= Statistical mean
σ	= Standard deviation

Subscripts

A	= Applied stress
p	= Propellant
PL	= Payload
R	= Resistive stress
s	= Structure
tu	= Ultimate stress
ty	= Yield stress
x,y,z	= Orthogonal axes

TECHNICAL PAPER

INHERENT CONSERVATISM IN DETERMINISTIC QUASI-STATIC STRUCTURAL ANALYSES

I. INTRODUCTION

In designing reliable and affordable next generation carriers to orbit, performance of aerospace frame structures once more emerges as a critical limiting component requiring innovative configurations, updated materials, and refined analyses. And because designing for structural safety is in direct contention with performance and cost, a study was initiated to explore the prevailing deterministic structural safety factor assumptions and standards for their often suspected conservatism.

The origin of the conventional safety factor, its simplistic application, its verification criteria, and launch vehicle performance sensitivities to its excesses were briefly reflected upon. Though raw data dispersions were noted to be processed through probability techniques, current practice is to reduce them to deterministic input values for quasi-static substructural loads and stress computations; and there lies the cradle of excessive conservatism. As deterministic values were combined and manipulated throughout the computations, the imbedded statistical dispersions were impelled to be summed rather than root-summed-squared, which violated error propagation laws that predicted cumulative excessively applied loads and stresses.

Conserving the data in statistical format and complying with the error propagation laws throughout the structural computation processes lead to designer controlled and leaner safety factors which are treated in the conventional manner to assess the structural relative safety and to experimentally verify the structural response. In examining the role of the safety factor in the failure concept, the safety factor only increases the number of standard deviations of the applied stress, which decreases the applied and resistive stress overlap.

The safety factor expressing the applied and resistive stresses in statistical format was integrated into the first order reliability method to provide the option of designing the structural frame to a specified uniform and absolute reliability, or to relate the arbitrarily specified design safety factor to a normalize reliability. Unexpectedly, the safety factor as currently applied is proportionately relative only with materials and welds having identical coefficients of variation. As the coefficient of variation increases, the relative proportion of the safety factor and the normalized reliability decreases.

II. CONVENTIONAL SAFETY FACTOR

The conventional safety factor has served the aerostructural community very well through many progressive changes in materials, associated disciplines, and subjective assumptions. Perhaps its success is owed to its unchallenged safety conservatism and to its neglected performance potential. It should be instructive to assess excessive safety factor consequences to payload performance, and subsequently to cost, through its sensitivity to surplus weight over global structural areas. Local stress concentration regions identified by abrupt changes of geometry, loads, metallurgy, and temperature are of less consequence to performance.

A. Historical Note

Though there was very little statistical data on loading conditions in 1932, there was evidence that successfully designed airplanes did not yield. Since the common structural materials were 17ST aluminum alloy and 4130 steel, and since the aluminum had an ultimate-to-yield stress ratio of 1.5, the arbitrary 1.5 safety factor at fracture was universally accepted[1] by the Commerce, Air Force, and Navy departments. Steel structures were obviously penalized by that standard, but it was acceptable because of their limited applications at that time. During the Apollo program, NASA field centers chose to reduce the ultimate safety factor to 1.4, in order to capitalize on recently improved aluminum properties to increase vehicle performance. Again its penalty to high strength steels was ignored. The 1.4 factor at fracture is now an official NASA standard,[2] and it is expressed by:

$$SF_{tu} = 1.4 = \frac{S_{tu}}{S_A} \ . \tag{1}$$

Though safety factors generally are specified at all levels of material fundamental property changes, the safety factor based on polycrystalline yield is difficult to verify. Plastic deformation starts in different locations, numbers, and intensities, and it is hard to detect and determine where and how much deformation has progressed until large enough parts have been affected and detected. This phenomenon explains why different gauge lengths in uniaxial tensile tests provide different elastic limits, why yield coefficients of variations are higher than strength variations, and why the elastic limit is more difficult to detect in brittle materials. Exceeding the yield point permanently changes the structural boundary conditions and reduces fatigue life. Similar degradation phenomena may exist in composites (matrix cracking) with varying consequence in stiffness or utility. These, among others, are reasons for which explicit yield safety factors should be contingent on the consequence[3] of each operational case.

Nevertheless, current deterministic experimental tests consist of a static structural elastic response verification to predicted maximum operational environments, and of an ultimate safety factor of 1.4 verification to avoid operating in the plastic region of most aerostructural polycrystalline materials. The fracture safety factor subsequently has been rationalized to cover rare operational events in which no

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statistical design data exists. Its traditional and historical usage now exerts the greatest influence on design and contractually binding acceptance criteria.

It should be cautioned that a deterministic static test is a pass-fail experiment of combined physical phenomena resulting in single numbers that often approximate expected fracture values for the wrong reasons. Comprehensive pretest and posttest analyses are essential for resolving the right reason and for learning the most from the test investment. A test not conducted to fracture provides little information no matter how well the complex structure performs thereafter. While the loading instant, location, and nature of yielding may be difficult to experimentally detect, testing to fracture leaves little doubt.

B. Consequence of Excessive Conservatism

A deterministic static test should prove the article to not be marginally or excessively safe for all the right reasons. A meaningful stress audit should present negative and positive margins and consequences for both cases. While the cause and consequences of negative margins derived from tests are invariably modified, sources and consequences of excessive positive margins are often ignored at the ultimate expense of payload performance. The dilemma and magnitude of excessive conservatism may be best appreciated through illustrations of the carrier performance sensitivities to surplus structural safety. For example, a monocoque shell structural weight is defined by:

$$W_s = C_1 rlt$$

where two of the dimensions (r and l) envelop the structural element size and shape that are usually optimized by the system's operational environments, payload performance, and cost. The thickness, t, is controlled by normal stress limitations and by the designer's selected safety. Applying the safety factor on the load, p, a hypothetical tank minimum thickness is approximated from the material physics:

$$t = \frac{p(SF)r}{S_{tu}} \quad ,$$

and the shell weight is rewritten to relate to the safety factor by:

$$W_s = \frac{C_1 lr^2 p(SF)}{S_{tu}} \quad .$$

The weight performance sensitivity[4] to the safety factor is given by the change in weight to change in the safety factor:

$$\frac{\partial W_s}{W_s} = \frac{C_1 lr^2 p \partial(SF) S_{tu}}{C_1 lr^2 p(SF) S_{tu}} = \frac{\partial(SF)}{(SF)} \quad ,$$

resulting in a direct proportionality of 1 percent increase in weight for each percent increase in safety factor. This sensitivity may be a useful thumb rule for assessing the safety factor penalty to structural element performance subjected to normal stresses.

The length and width of a plate in bending is another example of the element size that is fixed by the structural system, and the thickness parameter is approximated from:

$$t = \left[\frac{C_2 M(SF)}{b S_{tu}} \right]^{\frac{1}{2}} ,$$

where the ultimate tension and compression stresses are assumed symmetrical. The plate weight is:

$$W_s = C_3 lb \left[\frac{M(SF)}{b S_{tu}} \right]^{\frac{1}{2}} .$$

Proceeding as before, the bending plate weight sensitivity to the safety factor is the partial of the weight divided by the weight:

$$\frac{\partial W_s}{W_s} = 0.5 \frac{\partial (SF)}{(SF)} ,$$

and the bending sensitivity turns out to be half of the normal stress sensitivity. However, bending stresses are primarily local and are not as weight prevalent as normal stresses in aerospace structures. Sensitivities should be verified on all global, high-performance flight structures to effectively support trades and design insight.

The ultimate ripple effect of excessive safety factors may be realized from flight performance parameters. Using the well-known rocket equation:

$$\Delta V - \Delta Vloss = I_{sp} g \ln \frac{W_p}{W_s + W_p + W_{PL}} ,$$

and assuming the orbital and propulsion parameters are constant, then the mass fraction remains a constant:

$$exp. \frac{\Delta V - \Delta Vloss}{I_{sp} g} = \frac{W_p}{W_p + W_s + W_{PL}} = C_4 ,$$

and the propellant weight to orbit is:

$$W_p = \frac{C_4}{1 - C_4} (W_s + W_{PL}) .$$

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The sensitivity of the propellant weight increase to accommodate structural weight increase is:

$$\frac{\partial W_p}{W_p} = \frac{W_s}{W_s + W_{PL}} \frac{\partial W_s}{W_s} \quad .$$

Using the weight-to-safety factor relationship developed above, the sensitivity of increased propellant weight consumption to accommodate the safety factor increase is:

$$\frac{\partial W_p}{W_p} = \frac{W_s}{W_s + W_{PL}} \frac{\partial (SF)}{(SF)} \quad .$$

The ripple effect continues in that, increasing the propellant weight further increases the tank size and tank weight, which necessitates more propellant weight, etc. The increased tank size and associated propellant loading facilities represent the initial manufacturing costs. The increased tank and propellant weights are the recurring costs of lost performance. Recognizing the penalties of excessive safety factors and potential rippling effects, then it seems not enough for a senior structural analyst to design a reliable structure. His hallmark should be a lean reliable design such as to create and shift the least excessive conservatism burden downstream onto the vehicle performance and supporting disciplines.

On the other side of conservatism is the acceptance and compensation of marginal assumptions made in structural modeling. The above shell thickness example was a strength of materials approximation of an elasticity theory tube in which the thickness-to-radius ratio is assumed very small, such that the radial strain may be assumed uniform across the thickness to simplify the compatibility and boundary conditions. The error is often less than 1 percent for common shell properties and is usually ignored. The plate is also a strength of materials approximation which assumes the elastic cross section to remain rectangular after bending. Most analytical and finite element method models are also approximations to simplify and expedite solutions with negligible resulting errors. Nevertheless, it is incumbent on the analyst to assess and estimate errors before dismissing them or lavishing approximate solutions with arbitrary gut compensating factors. It's all part of the professional bag of creditable experience.

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III. DATA CHARACTERIZATION

Models are idealized into the simplest mathematical expressions within the physical phenomena of the data and its intended application. Not all data is equally important, as noted by Pareto's principal[5] and as can be distinguished by sensitivity analyses. Data having negligible effect on performance may be reduced to deterministic values. Data having major consequences must be characterized and processed in statistical format throughout the computational process involving single and combined phenomena.

Data that must be expressed and processed in statistical format is developed in this section, and those familiar with probability methods may wish to defer it. Those pragmatists interested in reviewing basic probability expressions will find most of the statistics and probabilistic techniques required for structures in this section.[5] While acknowledging the immense contributions of statisticians to structural analyses, it is often more important, in an established process, that structural analysts learn a little statistics than a statistician know a little structures.

A. Central Moments and Distributions

Variations are intrinsic in all observed phenomena and are of little engineering information in raw form. The best approach to summarizing a table of raw data of any distribution is to define the centroid about which the data is scattered. This variable is the first central moment[6] of the independent variables commonly known as the sample mean, or sample average, and is defined by:

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{2}$$

where x_i is the ith specimen value, and n is the total number of specimens. The sample mean is calculated from a limited sample size and is, therefore, an estimate of the population mean. A measure of the dispersion of the data about the mean is the second central moment known as the sample variance, and its square root:

$$\sigma = \left[\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu)^2 \right]^{\frac{1}{2}}, \tag{3}$$

is called the sample standard deviation "σ" and is a measure of the actual variation in a set of data.

The coefficient of variation is a relative variation, or scatter, among sets and is defined as the ratio of the standard deviation and the mean:

$$\eta = \frac{\sigma}{\mu} . \tag{4}$$

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The coefficient of variation is an effective technique for supporting judgment through comparison with other known events. Coefficients of variation are known to be small for biological phenomena, but they are large for natural materials. Coefficients of variation are small for highly controlled manmade materials and are larger for brittle materials. A knowledge of typical coefficients of recurring sources may serve as a source for judging quality and acceptability of data. Estimates of some common material structural properties characterized by coefficients of variations are metal ultimate strengths 0.05, yield 0.07, weights 0.015, steel fracture toughness 0.07, constant amplitude fatigue 0.40, and filament composite strength 0.12.

Another technique used to evaluate raw data is the population probability density distribution, which defines the area shape of the distribution. Distribution shapes are fixed by their natural scatter of data about their means. Shapes are modeled by distribution functions to estimate the probability of a desired value for an assigned range of probability. As shapes become more complex, distribution models become more difficult, and skills and labor to apply them escalate. There are obviously many continuous distributions that may be constructed and favored by academicians, but normal distributions are most widely used by practitioners because they are the easiest and because the mean of "n" independent observations is believed to approach a normal distribution as "n" approaches infinity (central limit theory). It is also a good representation of many natural physical variables or for small samples with no dominating variance. The equation of the normal probability density distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp - \frac{1}{2}\left[\frac{x_i - \mu}{\sigma}\right]^2, \text{ for } -\infty < x < \infty \ , \tag{5}$$

assuming true values of a very large sample size. Normally distributed phenomena are sometimes disguised as nonnormal when data samples are selected from casually broadened and unscreened sources. Most metallic mechanical properties are known to be normally distributed, though fatigue properties are not, as fatigue is presently understood.

Analytical advantages in using normal probability distributions are that they may be completely defined by two variables (μ and σ), and it is the most developed theory having many of their characteristics well established. The area within a specified number of standard deviations of a probability density plot represents the proportion of the data population captured. One standard deviation about the average of a normal distribution is calculated to capture 68.3 percent of the data tabulated as illustrated in figure 1. Two standard deviations include 95.5 percent of the data, and three standards include 99.7 percent.
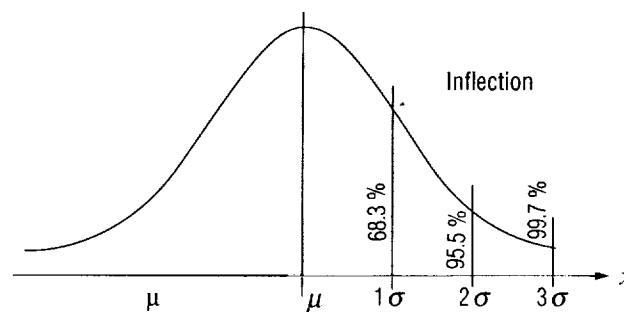

FIGURE 1.—Normal probability density plot.

The normal distribution plot and the mathematical test for the normality of data distribution are programmed in quick basic in appendices A1 and A2 for convenience. Most engineering data distributions are one-sided, occurring in the lower or upper sides of the data, such that either side may be developed into a split normal, should the distribution not pass the normality test. This cognizance and the central limit theory justify the presumption that structural probability demand and capability distributions may be normalized by constructing a mirror image of the engaged side about its peak frequency value and calculating the standard deviation from the constructed symmetrical distribution. This universal normalizing technique is illustrated in appendix A3 and is applicable to normal and nonnormal observed structural data. Hence, all structural data in this study is assumed to be generalized into normal probability distributions and, therefore, benefits from existing first-order techniques to simplify and expedite solutions with negligible consequences.

B. Statistical Format

In structural analysis, tolerance limit is a convenient statistical format for all input and output data which defines the distribution and specifies a statistical range of variations about the data's most probable value. Statistical tolerance limits may be defined from a normal probability density plot for any given proportion of data and are commonly expressed in statistical parameter format:

$$T_{lim} = \mu \pm N\sigma \ , \tag{6}$$

or in using equation (4):

$$T_{lim} = \mu(1 \pm N\eta) \ , \tag{7}$$

where T is the input-output data, such as loads or stress. The range factor N is a designer controlled number of standard deviations required to capture a specified percent of data from the high or low side ($\pm$) of a plot. On many nonredundant critical structures, the range factor of $N=3$ is selected to capture a realistically high percent or observed extreme of the data population. In current deterministic analysis, the single values of the mean and standard deviation are substituted in equations (6) or (7) to reduce them to single value input-output data. It must be emphasized that once the tolerance limit format is converted into a deterministic value, it cannot be decomposed into its original statistical format from the deterministic value alone.

However, true values of the mean and the standard deviation in equation (6) are not generally known from small sample sizes, for they may not contain a given portion of the population estimated by equations (2) and (3). In other words, the same test conducted on the same number of specimens by different experimenters will result in different means and standard deviations because of the inherent randomness in the specimens and testing. The population must contain results from all of these experiments.

To insure with a certain percentage of confidence that the given portion is contained in the population, a K-factor is determined to account for the sample size and proportion. Figure 2 provides the K-factor for random variables with 95 percent confidence levels and three probabilities (0.90, 0.95, and 0.99) in a one-sided normal distribution. Other confidence K-factors may be computed from the program provided in

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appendix B from which the K-factor maximum or minimum design value may be determined for a specified probability and confidence. That allowable design value is the upper or lower tolerance limit defined by:
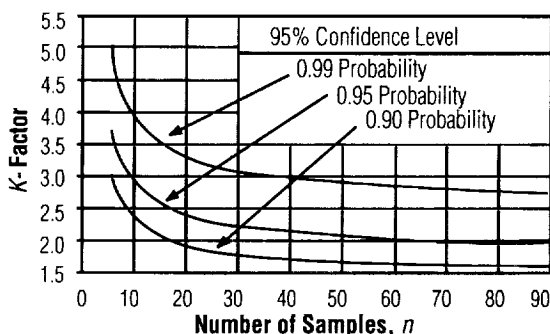
$$T_{lim} = \mu \pm K\sigma \quad .$$ (8)

FIGURE 2.—K-factors for one-sided normal distribution.

C. Resistive Uniaxial Stress

The resistive stress probabilistic distribution is a direct data characterization of material strength from a uniaxial stress test. While most of NASA material properties are specified by "A" and "B" bases, the tolerance limit of equation (8) is specified for the lower half of the distribution:

$$S_R = \mu_R - K\sigma_R \quad ,$$ (9)

and by the number of test samples. Test samples may range from standard uniaxial tensile specimens through pressure bottles. It's a trade between the initial cost of extensive material sample testing and the recurring cost of lost performance of global structures based on designing to a larger factor to compensate for small sample property predictions.

Consistent with critical main structures and welds, the stress dispersion is often autonomously assumed as 3 σ, or $K=3$, requiring at least 32 test samples (figure 2) for an A-Basis material. An A-Basis property allows that 99 percent of materials produced will exceed the specified value with 95 percent confidence. The B-Basis allows 90 percent with the same 95 percent confidence. Figure 3 illustrates the probability and confidence plot for an A-Basis design.
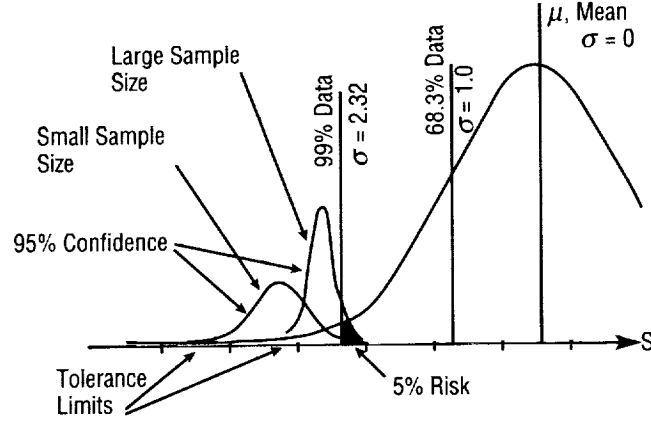
FIGURE 3.—One-sided normal distribution with A-Basis.

Most normally distributed material properties are developed in tolerance limit format as in equation (9). However, they are more often reduced and published as deterministic single values, and cannot be decomposed again into tolerance limit format as required for reliability analyses. These published[7] deterministic properties are a serious loss of existing but incompatible data for future applications of reliability methods.

D. Combining Statistical Data

Combining data that are statistically characterized variables and are mutually exclusive may be defined as a multivariable function by combining their dispersions through the following error propagation laws.[8,9] When two or more independent variables are added or subtracted, their means are added or subtracted and their standard deviations are root-sum-squared (rss) by the summation function rule:

$$\text{for } z = x \pm y, \quad \sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2} \ . \tag{10}$$

Applying this to the sum of a string of tolerance limits gives:

$$T = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \mu_i + \left[\sum_{i=1}^{n} N_i^2 \sigma_i^2 \right]^{\frac{1}{2}} \ . \tag{11}$$

When independent variables are multiplied and/or divided ($\pm$ exponent), their coefficients of variation are root-sum-squared according to the power function rule:

$$\text{for } z = x^a y^b, \quad \eta_z = \sqrt{a^2 \eta_x^2 + b^2 \eta_y^2} \ , \tag{12}$$

where η_z represents the coefficient of variation of that product. Elastic modulus and Poisson's ratio are defined by multivariables having measured dispersions and must be combined by the power function rule.

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Though these rules apply to all statistically formatted and manipulated input-output data through loads and stress computational processes, deviations are inaccessible for complying with these functional rules in deterministic methods.

E. Interfering Distributions

Another type of data characterization is the development of a third distribution from two opposing distributions defined by their tolerance limit formats, such as in the failure concept. Failure occurs when demand exceeds capability. When applied stresses and material strengths are defined by probability distributions, probability of failure increases as their tail overlap area increases as shown in figure 4. The overlap area suggests the probability that a weak material will encounter an excessively applied stress to cause failure. The probability of failure decreases as the designer controlled difference of the distribution means increases and the natural distribution shapes decrease.
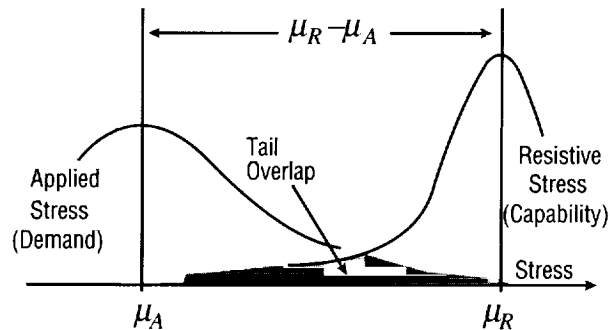
FIGURE 4.—Failure concept.

As discussed before, just the distribution side producing the worst case design problem is of any engineering interest, as is clearly demonstrated by the failure concept of figure 4. Only data from the right half of the applied stress distribution (greatest demand) is engaged with data from only the left side (weakest capability) of the resistive stress. Data from the other two disengaged distribution halves is irrelevant to the failure concept. Having defined the resistive stress distribution in subsection 3 above, the applied stress distribution computational process follows before the failure, or reliability, concept may be developed in section V.

IV. APPLIED UNIAXIAL STRESS

This section develops the normal distribution worst half of the uniaxial equivalent applied stress on a first-cut structural frame design, and then iterates allowable stresses to satisfy specified safety. It characterizes probability data in statistical format and scrutinizes all data input-output throughout structural computation processes for dispersions and assumptions contributing to excessive conservatism. Structural computational processes leading to applied uniaxial stress normal distribution include: multiaxial quasi-static design loads; multiaxial static design loads; multiaxial stresses; state of stress; failure criteria; applied uniaxial stresses.

A. Vehicle Quasi-Static Design Loads

Events that design vehicle substructures include on-pad assembly, liftoff, max Q, high-g, separation, etc. Launch vehicle forcing functions used to generate ascent generalized forces include: wind speed, shear, gust and direction; propulsion thrust rise, oscillations, and mismatch; thrust vector control angle and rate; vehicle acceleration and angle of attack; mass distribution; other special trajectory generated environments.

Because input environments to response analysis are time-dependent and statistically characterized, the induced loads output is also time-dependent and of a statistical nature. The response histories at select grid points are illustrated in figure 5, in which a specific time event may produce a maximum internal load for a degree of freedom (DOF) at one grid point only. Other time events produce maximum loads at other grid points as shown. Where a maximum internal load response is identified at a grid point, the free-body diagram of the included substructure experiencing that maximum response is constructed with all time-consistent loads acting along the total system. This computational process for designing different parts through time-consistent and statistically dispersed loads is repeated for each substructure at each unique event time producing the maximum load response about each axis. The end product of the structural response to environmental excitations is a set of maximum design loads, or "limit loads," and event times for all the system substructures and critical regions.
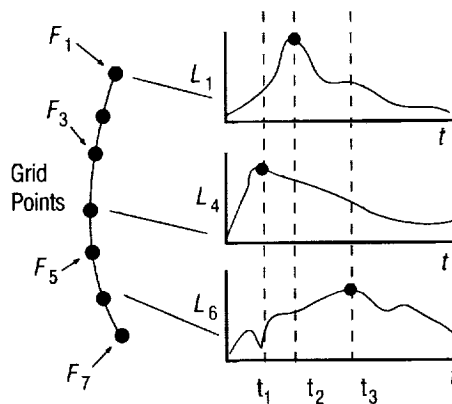

FIGURE 5.—Time-dependent response.

Aerospace loads modeling uses established computational structural dynamics principles and solution techniques[10,11] for multi degrees-of-freedom (MDOF) structures. Models assume the structural system to be represented by a network of finite elements designated along the body possessing mass, damping, and stiffness. Natural and induced environments act as forcing functions at discrete grid points. The motion of the total structure is composed of a system of substructures which are expressed by the linear matrix differential equation:

$$[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[K]\{X(t)\}=\{F(t)\} \ .$$

The acceleration matrix, $\{\ddot{X}(t)\}$, is the time-dependent physical coordinates at DOF. The $[M]$, $[C]$, and $[K]$ coefficients are mass, damping, and stiffness matrices, respectively. Dispersion of input data to these coefficients is usually constant or negligible and may be defined as $\sigma=0$ throughout the computations. The forcing function $\{F(t)\}$ is a matrix of time-dependent environmental excitations acting along the structural body. Its columns represent time increments. Its matrix rows represent discrete grid points of body internal DOF at which natural or induced environments are acting at one instant of time. Data characteristics of these forcing functions are of a statistical nature, having notable dispersions and consequences. Subsequently, they must be defined in statistical format and must be further treated by error propagation laws of equations (10) and (12).

However, these forcing functions are currently defined by deterministic variables and are bounded to be summed algebraically when combining through all the following major steps in the quasi-static computations. The combined summation includes the deviations which generate a major source of uncontrolled conservatism.

The above matrix differential equation is comprised of a set of coupled equations of motion which may be uncoupled through the mode-superposition method to determine the response of a system to a set of forcing functions. The system's undamped natural frequencies ω and mode shapes $[\phi]$ are solved from the undamped eigenvalue problem, $([K]-\omega^2[M])\ [\phi]=0$, to obtain the coordinate transformation:

$$\{X\}=[\phi]\{q\}= \sum_{r=1}^{N} \phi_r q_r(t) \ ,$$

where q is the generalized coordinates for $r=1, 2, 3, ...N$ modes. The shape matrix $[\phi]$ rows represent the mode shape values at each DOF grid point, and columns represent different mode shapes relating to each natural frequency. Substituting the coordinate transformation equation into the linear matrix differential equation and premultiplying by the transpose of the mode shape, results in the equation of motion in terms of modal matrices and generalized coordinates. Because of orthogonality, coefficient matrices are diagonal matrices, and the uncoupled system differential equation of motion reduces to:

$$[I]\{\ddot{q}(t)\}+[2\zeta\omega]\{\dot{q}(t)\}+[\omega^2]\{q(t)\}=[\phi]^T\{F(t)\} \ ,$$

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where $[I]=[\phi]^T[M][\phi]$, $[2\zeta\omega]=[\varphi]^T[C][\phi]$, and $[\omega^2]=[\phi]^T[K][\phi]$ are the generalized (unity) mass matrix, damping matrix, and stiffness matrix, respectively, in deterministic format, and $[\phi]^T\{F(t)\}$ is the generalized force in statistical format.

The generalized force is calculated from a given a set of forcing functions also in statistical format, and the generalized coordinates $\ddot{q},\dot{q},q$ are then determined by integrating the uncoupled system differential equation for events characterized by associated forcing functions. Substituting these generalized coordinates into the coordinate transformation equation above yields the desired system physical coordinates $\ddot{X},\dot{X},X$. These physical coordinates are then used to compute the substructure internal loads to form a set of quasi-static design loads calculated through a loads transformation matrix (LTM) of the inquired internal loads. Applying the substructure's stiffness matrix into the modal displacement method, the internal loads $\{L(t)\}$ of the substructure are given by:

$$\{L(t)\}=[\overline{K}]\{\overline{X}(t)\} \ ,$$

where $[\overline{K}]$ selects rows of the substructure stiffness matrix corresponding to the desired internal DOF grid points, and:

$$\{\overline{X}(t)\}=[T]\{X(t)\}$$

is the total substructure displacements. The $[T]$ matrix selects the substructure DOF out of the system displacements. The loads matrix may be rewritten as:

$$\{L(t)\}=[LTM]\{q(t)\} \ , \tag{13}$$

where:

$$[LTM]=[K] \ [T] \ [\phi]$$

is the load transformation matrix. The resulting internal loads, $\{L(t)\}$, are the desired quasi-static response load at grid point "g" substructure, and results are expressed with forcing functions F_i in statistical format and "c_i" time consistent response gains (influence coefficients):

$$L_g=c_1F_1+c_2F_2+c_3F_3+c_4F_4+c_5F_5+\text{K} \tag{14}$$

or:

$$L_g=c_1(\mu_1+N_1\sigma_1)+c_2(\mu_2+N_2\sigma_2)+c_3(\mu_3+N_3\sigma_3)+c_4(\mu_4+N_4\sigma_4)+\text{K} \tag{15}$$

influencing respective subscript grid point. The resulting equation (15) defines a linear combination of the elements of a random vector having a combined mean:

$$\mu_g=\frac{1}{\sum\limits_{i=1}^{n}c_i}\sum\limits_{i=1}^{n}c_i\mu_i \ , \tag{16}$$

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and combined standard deviation:

$$\sigma_g = \frac{1}{\sum\limits_{i=1}^{n} c_i N_i} \left[\sum_{i=1}^{n} (c_i N_i \sigma_i)^2 \right]^{\frac{1}{2}} .$$ (17)

Combining equations (16) and (17), and autonomously selecting the final probability range factor N, the desired output probabilistic response (or limit) load at grid point g is:

$$L_g = \mu_g + N\sigma_g .$$ (18)

However, in the deterministic method, all the terms in equation (15) are summed which reduces the deterministic load response to:

$$L_g = \sum_{i=1}^{n} c_i \mu_i + \sum_{i=1}^{n} c_i N_i \sigma_i .$$ (19)

While the first term on the right side of equation (19) may be reduced to the combined mean of equation (16), the second term violates the summation function rule, equation (11), and reflects the worse-on-worse input-output process which is excessively conservative.

B. Total Combined Structural Loads

Other structural loads that must be combined with the quasi-static loads are the static ground and flight environmentally induced loads which include thermal, pressure, vibration, acoustic, etc. These loads are statistically derived and must be statistically formatted as in equation (6) and combined consistently with operational event and time by equation (11):

$$L_g = \sum_{k=1}^{n} \mu_k + \frac{N_g}{\sum\limits_{k=1}^{n} N_k} \sum_{k=1}^{n} N_k \sigma_k ,$$ (20)

or:

$$L_g = \mu_g (1 + N_g \eta_g) ,$$ (21)

where L_g is the load at grid point g, and k represents all the induced loads at an event and time. N_g is autonomously selected for that grid point. Induced loads with negligible variations and consequences may simplify the computations by assuming $\sigma = 0$ in the summation function rule.

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However, in the current deterministic method, all static loads are reduced to deterministic terms and added to deterministically reduced quasi-static loads in which all means and standard deviations are added:

$$L_g = \sum_{k=1}^{n} \mu_k + \sum_{k=1}^{n} N_k \sigma_k \quad , \tag{22}$$

contrary to the summation function rule of equation (11). Furthermore, the statistically formatted static load cannot be appropriately added to the deterministic quasi-static load. This step in the loads process generates a second source of excessive conservatism, in which the correct total load derived from equation (20) is smaller than the determinist total load from equation (22).

C. Structural Stress Response

The structural stress model is represented by the same network of finite elements designated along the body with grid points corresponding with those on the quasi-static loads model above. Then each substructure is analyzed for the worst combination of time constant loads acting over the grid points that produce the greatest applied stress. Since input loads are expressed in statistical format as in equation (21), the computed stress output should also be produced in statistical format:

$$S = \mu_g (1 + N_g \eta_g) \quad , \tag{23}$$

through the error propagation laws of equations (10) and (12) for all stress components at each grid point. The coefficient of variation η_g is the same as that derived from loads in equation (21).

In deterministic methods, the input statistical variables, equation (21), are treated as single values, and they again violate the error propagation laws when combined in computations of the structural response. To create a third source of excessive conservatism the NESSUS/FEM[12] module should be considered when using finite element methods.

D. Equivalent Uniaxial Strength

Proceeding with the search for sources of excess conservatism in the current deterministic process, the state of stress and failure criteria were examined. In order that applied triaxial stresses acting at any grid point, may be equated to the resistive (or ultimate) uniaxial stress in equation (1), the applied triaxial stresses must first be reduced into one dimensional (resultant) stress and then indexed to an equivalent uniaxial yield strength.

The complex state of stress at a point on an oblique surface of a solid may be readily derived[13] by modeling the three normal principal stress components acting along the orthogonal principal axes of a tetrahedron. The sum of forces along each axis provides three linear homogeneous equations to be solved simultaneously. A nontrivial solution of stress on the oblique surface is obtained by setting the resulting determinant of the stress coefficients to zero. The solution to the determinant is reduced to a cubic equation having three combinations of component stresses as coefficients I_i of the oblique normal stress:

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$$S^3 = I_1 S^2 - I_2 S - I_3 = 0 \quad ,$$

known as invariant. The first invariant is the sum of the determinant diagonal which relates to the hydrostatic stress:

$$I_1 = S_1 + S_2 + S_3 \quad ,$$

with a mean stress of $S_{mean} = I_1/3$. The second invariant is the sum of the principal minors:

$$I_2 = \tfrac{1}{2} \left[(S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2 \right] \quad ,$$

that relates to shear stress. These shear stresses should be in statistical format but are currently defined as deterministic values. The third invariant is the determinant of the whole matrix. These invariants of the state of stress are noted to be independent of material properties, which incites the next point.

Currently, there is no theory that directly relates multiaxial stresses with uniaxial yield or ultimate stress. However, there are several criteria in which the elastic limit of a multiaxial stress state is empirically related to the uniaxial tensile yielding, and results are reasonably consistent with experimental observations. The Mises yield criterion[14] is based on the minimum strain energy distortion theory which supposes that hydrostatic strain (change in volume) does not cause yielding, but changing shape (shear strain) does cause permanent deformation. Hence, the yield criterion relates the experimental uniaxial tensile elastic limit, S_{ty}, to the principal shear stresses through the square root of only the second invariant of the stress matrix. Then using the second invariant, the Mises initiation of yield criterion is expressed in its familiar form by:

$$S_{ty} = \frac{1}{\sqrt{2}} \left[(S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2 \right]^{\frac{1}{2}} \quad , \tag{24}$$

which depends on a function of all three principal shear stresses. Because of squared terms, it is independent of stress signs and, therefore, it is applicable to compression and tensile combinations of multiaxial stresses. And because of isotropy, the second invariant implies that it is independent of selected axes and may be expressed about any oblique plane:

$$S_{ty} = \left[S_x^2 + S_y^2 + S_z^2 - S_x S_y - S_x S_z - S_y S_z + 3(S_{xy}^2 + S_{xz}^2 + S_{yz}^2) \right]^{\frac{1}{2}} \quad . \tag{25}$$

Using equation (25), the pure shear yield stress reduces to $S_{sy} = \dfrac{S_{ty}}{\sqrt{3}}$ and is a good approximation of test results. Having established the yield stress by equations (24) and (25), the criterion also expresses the equivalent applied uniaxial tensile stress over the total elastic and inelastic range about that yield stress:

$$S_{eqiv.} = \left[S_x^2 + S_y^2 + S_z^2 - S_x S_y - S_x S_z - S_y S_z + 3(S_{xy}^2 + S_{xz}^2 + S_{yz}^2) \right]^{\frac{1}{2}} . \tag{26}$$

As noted above, these invariant stresses are in fact probabilistic stresses. But the current deterministic application of the Mises criterion, as expressed by equation (26), again violates the error propagation laws enforcing a larger combined stress case than the statistically complied case, thus rendering a fourth source of excessive conservatism. Since the four sources of excessive conservatism build on each other sequentially in the applied stress computational process, equations (25) and (26) represent the total accumulative yield and allowable stresses respectively. If the Mises stress of equation (25) exceeds the allowable stress of equation (1), the structural thickness is increased and the applied stress process is iterated.

Returning to the Mises criterion of equation (26), the local multiaxial stresses should be in statistical format:

$$S_i = (\mu_i + N_i \sigma_i) \ , \tag{27}$$

and may be appropriately combined through the error propagation laws by expanding the functional relationship in a multivariable Taylor series about a design point (mean) of the system. The mean of the Mises combined applied stresses is determined from equations (26) and (27):

$$\mu_A = \left[\mu_x^2 + \mu_y^2 + \mu_z^2 - \mu_x \mu_y - \mu_y \mu_z - \mu_z \mu_x + 3(\mu_{xy}^2 + \mu_{yz}^2 + \mu_{zx}^2) \right]^{\frac{1}{2}} . \tag{28}$$

The combined standard deviation is calculated from:

$$\sigma_A = \left[\left(\frac{\partial S_A}{\partial S_x} \sigma_x \right)^2 + \left(\frac{\partial S_A}{\partial S_y} \sigma_y \right)^2 + \left(\frac{\partial S_A}{\partial S_z} \sigma_z \right)^2 \right.$$

$$\left. + 9 \left\{ \left(\frac{\partial S_A}{\partial S_{xy}} \sigma_{xy} \right)^2 + \left(\frac{\partial S_A}{\partial S_{yz}} \sigma_{yz} \right)^2 + \left(\frac{\partial S_A}{\partial S_{zx}} \sigma_{zx} \right)^2 \right\} \right]^{\frac{1}{2}} , \tag{29}$$

and the controlled standard deviation is:

$$\dot{\sigma}_A = \left[\left(\frac{\partial S_A}{\partial S_x} N_x \sigma_x \right)^2 + \left(\frac{\partial S_A}{\partial S_y} N_y \sigma_y \right)^2 + \left(\frac{\partial S_A}{\partial S_z} N_z \sigma_z \right)^2 \right.$$

$$\left. + 9 \left\{ \left(\frac{\partial S_A}{\partial S_{xy}} N_{xy} \sigma_{xy} \right)^2 + \left(\frac{\partial S_A}{\partial S_{yz}} N_{yz} \sigma_{yz} \right)^2 + \left(\frac{\partial S_A}{\partial S_{zx}} N_{zx} \sigma_{zx} \right)^2 \right\} \right]^{\frac{1}{2}} . \qquad (30)$$

The probability range factor is calculated from equations (29) and (30):

$$N_A = \frac{\dot{\sigma}_A}{\sigma_A} , \qquad (31)$$

and using equation (6), the coefficient of variation is:

$$\eta_A = \frac{\sigma_A}{\mu_A} . \qquad (32)$$

The partials of each term under the radical of equation (26) are given by the chain rule:

$$\frac{d \sqrt{w(S_i)}}{dS_i} = \frac{d\sqrt{w}}{dw} \frac{dw}{dS_i} = \frac{1}{2\sqrt{w}} \frac{dw}{dS_i} .$$

The normal partials are:

$$\frac{\partial S_A}{\partial S_x} = \frac{2\mu_x - \mu_y - \mu_z}{2 S_A} , \quad \frac{\partial S_A}{\partial S_y} = \frac{2\mu_y - \mu_x - \mu_z}{2 S_A} , \quad \frac{\partial S_A}{\partial S_z} = \frac{2\mu_z - \mu_y - \mu_x}{2 S_A} , \qquad (33)$$

and the shear partials are:

$$\frac{\partial S_A}{\partial S_{xy}} = \frac{3\mu_{xy}}{S_A} , \quad \frac{\partial S_A}{\partial S_{yz}} = \frac{3\mu_{yz}}{S_A} , \quad \frac{\partial S_A}{\partial S_{zx}} = \frac{3\mu_{zx}}{S_A} . \qquad (34)$$

All partials are evaluated at the system mean. Applying equations (28), (29), (22), and (23) provides the appropriate applied stress of the system in statistical format:

$$\dot{S}_A = \mu_A (1 + N_A \eta_A) . \qquad (35)$$

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V. SAFETY CHARACTERIZATION

Having defined the probabilistic resistive stress by equation (9), and having amended the probabilistic applied stress by equation (35), a relative probabilistic safety factor may be established, and its experimental verification limits, its relative role in the failure concept and absolute safety index may be completely characterized.

A. Probabilistic Safety Factor

Generically, a safety factor is expressed as the ratio of the resistive to the applied stresses, where the resistive stress is reference to the ultimate or yield strength of an A- or B-based material and the applied stress is designed to not exceed an arbitrarily specified safety factor. A probabilistic safety factor expresses the applied and resistive stresses in probabilistic format:

$$SF = \frac{S_R}{S_A} = \frac{\mu_{tu}(1 - K\eta_R)}{\mu_A(1 + N_A \eta_A)} \quad . \tag{36}$$

In designing to a specified safety factor, the designer controllable variables are the: material K-factor by choosing the number of test specimens as discussed before; the applied stress range factor selected autonomously by the loads analysts; and maximum predicted applied stress to not exceed the specified safety factor.

To verify the design safety factor of equation (36) on a substructure, two conditions must be experimentally confirmed, the maximum predicted operational applied stress and the structural response to the predicted applied stress. It must be noted that the maximum predicted operational applied stress can only be confirmed by field or flight tests, and it may be many flights before each unique event producing the maximum load environment at each different substructure is achieved. It should be further noted that if the design operational applied stress is exceeded well into the program operational phase flight (exceeds yield safety factor), the contingency safety factor of equation (1) will accommodate it as nonlinearly inelastic on the "first loading" (cycle) and subsequently linearly elastic.

The structural response of the NASA standard is experimentally verified through a static test in which the maximum predicted operational applied stress with a safety factor of at least one will cause the material to yield, and the operational applied stress increased by a safety factor of at least 1.4 times will produce fracture. If yield and fracture occur prematurely, the stress math model or material properties are in error, or a sneak phenomenon may be involved. Response dispersions of test articles vary with materials and manufacturing dispersions.

The probabilistic structural response of equation (36) is experimentally verified to a predicted leaner applied stress. Structures tested to the more conservative deterministic applied stresses possess more margins

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than the more appropriate probabilistic applied stresses . Therefore, it is expected that those existing structures tested to deterministic stresses and found submarginal may be perfectly adequate. Those found adequate may be excessively conservative.

B. Failure Concept

While the prevailing deterministic safety factor is the most commonly specified structural safety criterion, the mechanics of how safety is achieved in the failure concept are not expansively understood. In applying the probabilistic safety factor to the failure concept diagram of figure 6, equation (36) is noted to define three zones:

$$\mu_R - S_R = \mu_R - (\mu_R - K\sigma_R) = K\sigma_R \ , \tag{37}$$

$$S_R - S_A = (SF)S_A - S_A = (SF-1)S_A = (SF-1)(\mu_A + N_A\sigma_A) \ , \tag{38}$$

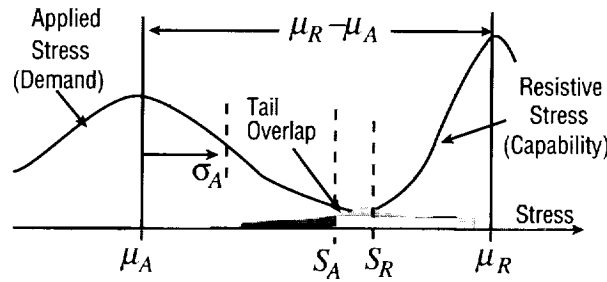$$S_A - \mu_A = (\mu_A + N_A\sigma_A) - \mu_A = N_A\sigma_A \ . \tag{39}$$

FIGURE 6.—Failure concept governing zones.

The mid zone is observed as an extension of the applied stress distribution by the safety factor:

$$S_R - S_A = \left(1 - \frac{1}{SF}\right)S_R \ . \tag{40}$$

Thus, the primary role of the safety factor is to decrease the applied stress tail overlap in the failure concept by extending the applied stress tolerance limit in figure 6 through the combined equations (38) and (39):

$$N_{eff}\sigma_A = S_R - \mu_A = N_A\sigma_A + (SF-1)(\mu_A + N_A\sigma_A) \ ,$$

from which the effective applied stress range factor is:

$$N_{eff} = \frac{SF-1}{\eta_A} + N_A SF \ . \tag{41}$$

The sum of the three zones is the difference of the applied and resistive stress distribution means:

$$\mu_R - \mu_A = (\mu_R - S_R) + (S_R - S_A) + (S_A - \mu_A) \quad . \tag{42}$$

Though equation (42) combines probability contributions of applied, resistive, and safety factor range factors which together decrease the tails overlap, and failure as shown in figure 6, their integrated difference suggests a relative safety assessment looking for an absolute safety reference.

C. First Order Reliability

Many advanced techniques are being investigated and are evolving[15] for providing reliable structural designs. In the meantime, assuming split normal probability distributions and defining resistive and applied stresses in probabilistic format as in equation (36) leads to the first order reliability method may be compatible with prevailing practices, codes, and skills.

The concept of failure was introduced by figure 4, in which the distribution tails overlap suggests the probability that a weak resistive material will encounter an excessively applied stress to cause failure. This is to say that the probability of success is reliability and that the reliability is less than 100 percent. Therefore, the probability of interference is the probability of failure and is governed by the difference of their means, $\mu_R - \mu_A$. Increasing the difference of the means decreases the tail interference area.

Given that the applied and resistive stress probability density functions are independent, they may be combined to form a third random variable density function[16] in $y = S_R - S_A$. If S_R and S_A are normally distributed random variables, then $y = S - S_A$ are also normally distributed and:

$$P_y = \frac{1}{\sigma_y \sqrt{2\pi}} EXP\left[-\frac{1}{2}\left[\frac{y - \mu_y}{\sigma_y} \right] \right] \quad , \tag{43}$$

where $\mu_y = \mu_R - \mu_A$ and $\sigma_y = \sqrt{\sigma_R^2 + \sigma_A^2}$. The y-variable distribution is plotted in figure 7.
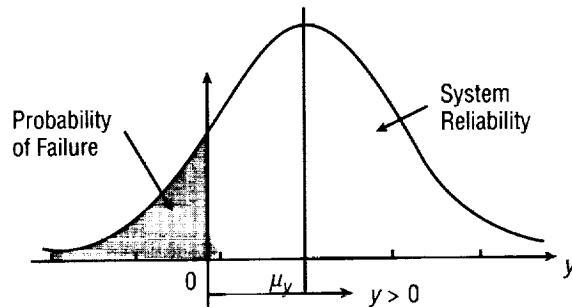

FIGURE 7.—Density function of random variable y.

The reliability of the third density function expressed in terms of y is:

$$R = P(S_R > S_A) = P(y > 0) = \int_0^\infty P_y \, dy \ ,$$

(44)

where P_y is the y-density function of equation (43). Letting $Z = \dfrac{y - \mu_y}{\sigma_y}$, then $\sigma_y dz = dy$. The lower limit of Z is:

$$Z_l = \frac{0 - \mu_y}{\sigma_y} = -\frac{\mu_R - \mu_A}{\sqrt{\sigma_R^2 + \sigma_A^2}} \ .$$

As y approaches infinity, Z approaches infinity, and the reliability of equation (44) is reduced to:

$$R = \frac{1}{\sqrt{2\pi}} \int \left[exp\left(-\frac{Z^2}{2} \right) \right] dZ \ .$$

(45)

The integration of equation (45) is programmed in the appendix C. Given the reliability R, the safety index "Z" value is printed, which may then be translated into statistical design parameters through the safety index expression:

$$Z = -Z_l = \frac{\mu_R - \mu_A}{\sqrt{\sigma_R^2 + \sigma_A^2}} \ .$$

(46)

Equation (46) formulates the probability concept. Increasing the safety index and the standard deviations increases the means difference, which decreases tail interference area and the probability of failure. The reliability relationship with the safety index is plotted in figure 8, and the reliability notation 0.9_n represents n 9s after the decimal.
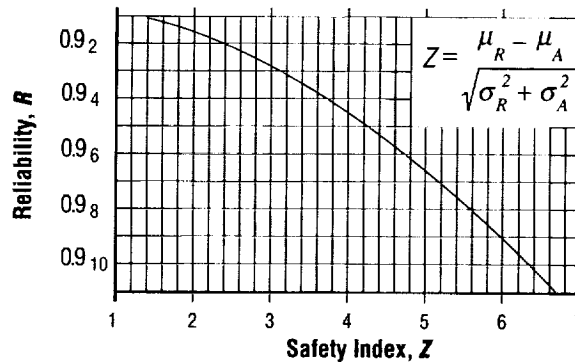

FIGURE 8.—Reliability versus safety index.

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Recognizing that the safety index from equation (46) shares the same difference and function of the applied and resistive stress distribution means expressed by equation (42), then substituting and simplifying reduces the safety index to the desired expression:

$$(47)$$

The designer autonomously selected variables[17] are the safety factor and the range factors. The safety index is noted to be about an order of magnitude more sensitive[18] to the safety factor:

$$\frac{\partial Z}{Z} = C_6 (1 + N_A \eta_A) \frac{\partial (SF)}{(SF)} \quad,$$

than to the range factor:

$$\frac{\partial Z}{Z} = C_6 N_A \eta_A \frac{\partial N_A}{N_A} \quad,$$

where the common coefficient is $C_6 = \frac{(SF)\mu_A}{\mu_R - \mu_A}$. The safety index is even less sensitive to coefficients of variation. The coefficients of variation are passive variables derived from statistical data, and they vary and combine in the structural computational process from region to region with variations in applied stresses and materials (including welds), which suggests that the safety factor or safety index varies from region to region. A specified common safety factor will not provide a common reliability.

The safety index provides structural analysts the options of designing substructures to specified uniform safety factors and then calculating the absolute reliability from region to region having different statistical variables, or of designing substructures to specified uniform reliability by adjusting safety factors from region to region. Equation (47) is particularly important to the structural system in that the combined probability of range and safety factors autonomously specified by the materials, loads, and stress disciplines may be integrated and optimized into a specified absolute reliability uniformly across the vehicle structure regardless of region to region material or operational environmental changes.

Developing a reliability criterion is an on-going concern in the aerostructural community. Selecting an arbitrary standard reliability is no better than the current standard safety factor. It should be expected to be derived from some compelling physical or economic constraint, such as risk:

$$risk = \sum_k P_k C_k \quad,$$

where P_k is the probability of the outcome and C_k is the consequence of the outcome k. However, risk requires great efforts, skills, and data. The results of risk are too sensitive to simple changes in failure processes, assumptions, and perceptions. A component common reliability criteria development is more compatible with the general designer culture and current practices.

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VI. CONCLUSIONS AND RECOMMENDATIONS

The cause and nature of the long-suspected excessive conservatism in the prevailing structural deterministic method have been identified as an inherent violation of the error propagation laws incurred when reducing statistical data to deterministic values and combining them through several structural computation processes. These errors are restricted to the applied loads and stress distribution computations, and because mean and variations of the tolerance limit format are added, the errors are positive, serially cumulative, and excessively conservative. Since the quasi-static structural deterministic conservatism varies with design and analysis and is generally indefinable, stress audits based on deterministic pass-fail safety factors are speculative, all of which should provide incentives for adapting probabilistic structural methods.

While most probabilistic methods in development would circumvent errors of propagation laws, patching, and partially converting existing deterministic methods may be more expedient and would provide the familiarity, confidence, and correlation with current experience base desired in new approaches.

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APPENDICES

The following programs are presented for the structural analysts' information, convenience, and library. Recognizing that programs are computer and software specific, the following are coded in Quick-Basic for their simplicity and application to pocket computers, and because of their easy conversion to other languages. This appendix is broken down as follows:

- A1. Normal probability density distribution program
- A2. Normality distribution test program
- A3. Normalizing skewed distribution (Split normal)
- B. *K*-factor program
- C. Safety index programs
- D. Mises criterion program

A1. Normal Probability Density Distribution Program

```
' NORMAL PROBABILITY DENSITY CURVE
OPEN"CLIP:"FOR OUTPUT AS #1
INPUT "MEAN =",M
INPUT "STD DEVIATION =",SD
INPUT "START =",XS
INPUT "FINISH =",XF
INPUT "INCREMENTS =",NX
DX=(XF-XS)/(NX-1)
  FOR I=1 TO NX
  X=XS + (I-1)*DX
  F=EXP(-.5*((X-M)/SD)^2)
  F=F/((2*3.14159*SD)^.5)
WRITE #3,X,F
PRINT X,F
NEXT I
CLOSE #1
STOP
```

A2. Normality Distribution Test Program

```
'  NORMAL DISTRIBUTION TEST
'  Kolmogorov-Smirnov (normality test)
'  Critical values (n > 30): a=.10, d=.805;
'     a=.05, d=.886; a=.01, d=1.03
OPEN"CLIP:"FOR OUTPUT AS #2
```

```
'  Input data
CLEAR:INPUT "N=";N
DIM  A(N),D(N),Z(N)
FOR I=1 TO N
INPUT A(I)
NEXT I

'  sort data
K=N-1
LINE180:FOR X=1 TO K
B=A(X)
IF B<=A(X+1) GOTO line250
A(X)=A(X+1)
A(X+1)=B
Y=1
T=X-1
line250:NEXT X
IF Y=0 GOTO line300
Y=0
K=T
GOTO LINE180
line300:
PRINT "SORT DONE"

'mean and std. deviation
FOR I=1 TO N
C=C+A(I)
D=D+A(I)*A(I)
```

```
NEXT I
M=C/N
SD=((D-N*M*M)/(N-1))^.5
PRINT "MEAN=";M
PRINT "STD DEV=";SD

'standardized normal
FOR I=1 TO N
Z(I)=(A(I)-M)/SD
NEXT I

'cumulative normal
FOR I=1 TO N
X=Z(I):T=X
G=EXP(-X*X/2)/SQR(2*3.14159)
A1=.31938:A2=-.35656:A3=1.78147
A4=-1.82125:A5=1.330427
IF X<0 THEN T=-X
Y=1/(1+.2316419*T)
P=(((((A5*Y+A4)*Y+A3)*Y+A2)*Y+A1)*Y
F=1-G*P
IF X<0 THEN F=1-F
DI=I/N          'empirical cumulative
D(I)=ABS (F-DI)
IF DM<D(I) THEN DM=D(I):J=I
NEXT I

'results
FOR I=1 TO N
PRINT D(I)
NEXT I
PRINT "WORST SAMPLE #";J
PRINT "ABS DIFFERENCE, D";DM
CLOSE #2
END
```

Figure A1 is a plot of the D critical values for a one-sided distribution. The distribution is not normal if the program test result exceeds the "D" critical value. Most engineering data distributions are one-sided, occurring in the lower or upper sides.
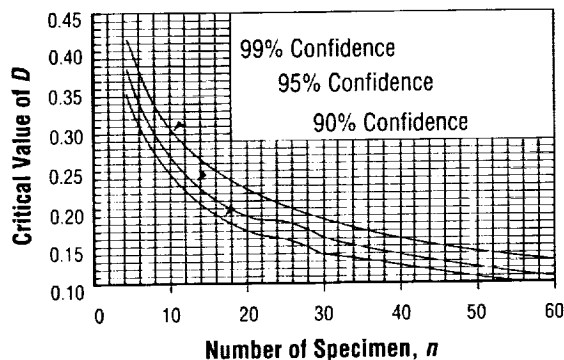

FIGURE A1.—One-sided test critical value of D.

A3. Normalizing Skewed Distribution (Split Normal)

Stress data are assumed to be based on a series of observed measurements reduced into a frequency distribution, or probability histograms, shown in figure A2. The base of the histogram is bounded by successive and equal ranges of measured values, and the heights represent the number of observations (frequency) in each range.
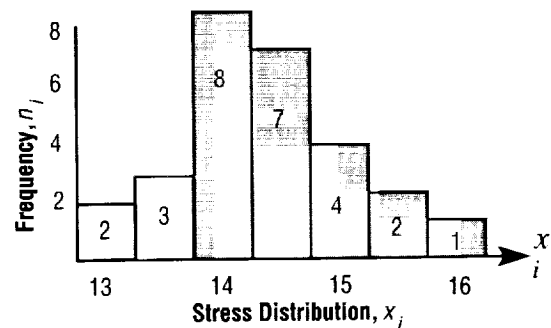

FIGURE A2.—Stress frequency distributions.

To illustrate the direct normalization of a skewed distribution, the stress frequency distribution data of figure A2 is applied to equations (2) through (5). Because the greater stress side defines the worst demand case (applied stress), only data from the shaded right side is used to calculate the normalized distribution variables. The distribution may be normalized by constructing a mirror image of the engaged side about its peak frequency value and calculating the standard deviation from the constructed symmetrical distribution.

The peak frequency from figure A2 distribution is the mean, $\mu = 14$ ksi.

Sample size is $\sum n = -n_1 + 2\sum n_i$
$$= 8 + 2 (8 + 7 + 4 + 2 + 1) = 36.$$

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Sum of variations about the mean

$$v=2\sum n_i(x_i-\mu)^2$$

$2\times8\ (14.0-14.0)^2=0$

$2\times7\ (14.5-14.0)^2=3.5$

$2\times4\ (15.0-14.0)^2=8.0$

$2\times2\ (15.5-14.0)^2=9.0$

$2\times1\ (16.0-14.0)^2=8.0$

$$v=28.5.$$

The variance is

$$\sigma^2=\frac{v}{\sum n-1}=\frac{28.5}{35}=0.81\quad,$$

and the standard deviation from equation (3) is $\sigma=0.90$ ksi.

The coefficient of variation from equation (40)

is $$\eta=\frac{\sigma}{\mu}=\frac{0.90}{14}=0.065\quad.$$

```
'SPLIT NORMAL DISTRIBUTION
'INPUT DATA
CLEAR: INPUT "PEAK FREQUENCY=",MU
INPUT "NUMBER OF BARS=",N
DIM F(N), X(N)
FOR I=1 TO N
INPUT F(I)
NEXT I
FOR I=1 TO N
INPUT X(I)
NEXT I

'CACULATE
S=0:FOR I=2 TO N
S=S+2 *F(I)
NEXT I
SS=S+F(1)
PRINT"SAMPLE SIZE=",SS
VMU=0:FOR I=1 TO N
VMU=VMU+2*F(I)*(X(I)-MU)^2
NEXT I
PRINT"VARIATIONS FROM MEAN=",VMU
PRINT "MEAN=",MU
SD=(VMU/(SS-1))^.5
PRINT "STD DEV=",SD
COF=SD/MU
PRINT"COEF OF VAR=",COF
```

B. *K*-Factor Program

```
'K- FACTOR
MARIO:
DEFDBL A-Z
INPUT"SAMPLE SIZE=";NS
INPUT "PROPORTION";P
INPUT "CONFIDENCE=";CL
IF NS>90 THEN PRINT"SAMPLE SIZE SHOULD
BE SMALLER THAN 90":WHILE INKEY$="":WEND
START=TIMER
PI=3.141592654#
'INVERSE NORMAL
Q=1-P:T=SQR(-2*LOG(Q))
A0=2.30753:A1=.27061:B1=.99229:B2=.0481
NU=A0+A1*T:DE=1+B1*T+B2*T*T
X=T-NU/DE
L0: Z=1/SQR(2*PI)*EXP(-X*X/2):IF X>2 GOTO L3
V=25-13*X*X
FOR N=11 TO 0 STEP-1
U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V
V=U:NEXT N
F=.5-Z*X/V
W=Q-F:GOTO L2
L3:V=X+30
FOR N=29 TO 1 STEP-1
U=X+N/V
V=U :NEXT N
F=Z/V :W=Q-F :GOTO L2
L2:L=L+1
R=X:X=X-W/Z
E=ABS(R-X)
IF E>.00001 GOTO L0
'END OF INVERSE NORMAL

'CALCULATION OF FACTORIAL
N=NS:NU=N-1
MT=INT(NU/2):UT=NU-2*MT
GT=1
FOR I=1 TO MT-1+UT
KT=I
IF UT=0 GOTO L1
KT=I-.5
L1:GT=GT*KT
NEXT I
GT=GT*(1+UT*(SQR(PI)-1))
GF=GT*2^(NU/2-1)
'END OF FACTORIAL

'SECANT METHOD
KP=X:J=1:K=KP
K0=K:GOSUB INTEGRATION:SF0=SF
K=K*(1+.0001):K1=K:GOSUB
INTEGRATION:SF1=SF
```

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```
BEGIN:K=K1-SF1*(K1-K0)/(SF1-SF0)
IF ABS((K1-K)/K1)<.000001 GOTO RESULT
J=J+1:K0=K1:K1=K:SF0=SF1
GOSUB INTEGRATION:SF1=SF:GOTO BEGIN
RESULT:FINISH=TIMER
BEEP:BEEP
PRINT "K =";USING"##.####";K
PRINT "TIME=";FINISH-START;"SECONDS"
'END OF SECANT METHOD
WHILE MOUSE(0)<>(1):WEND
GOTO MARIO

INTEGRATION:L1=0:L2=10
IF N>40 THEN L2=20
DL=KP*SQR(N):TP=K*SQR(N)
Y=NU/2
M=2:E=0:H=(L2-L1)/2
X=L1:GOSUB FUNCTION
Y0=Y:X=L2:GOSUB FUNCTION
YN=Y:X=L1+H:GOSUB FUNCTION
U=Y:S=(Y0+YN+4*U)*H/3
START:M=2*M
D=S:H=H/2:E=E+U:U=0
FOR I=1 TO M/2
X=L1+H*(2*I-1):GOSUB FUNCTION
U=U+Y
NEXT I
S=(Y0+YN+4*U+2*E)*H/3
IF ABS((S-D)/D)>.00001# GOTO START
SF=S/GF-CL
RETURN
'END OF SIMPSON

FUNCTION: Z=TP*X/SQR(NU)-DL
T0=Z:G0=1/SQR(2*PI)*EXP(-Z*Z/2)
A1=.3193815:A2=-.3565638:A3=1.781478:
A4=-1.821256:A5=1.330274
IF Z<0 THEN T0=-Z
W=1/(1+.231649*T0)
P1=((((A5*W+A4)*W+A3)*W+A2)*W+A1)*W
PH=1-G0*P1
IF Z<0 THEN PH=1-PH
Y=PH*X^(NU-1)*EXP(-X*X/2)
RETURN
```

C. Safety Index Programs

```
'SAFETY INDEX FROM RELIABILITY

'NORMIN (.5,P,1)
DEFDBL A-Z
LL: INPUT"Probability=";P
PI=3.141593
```

```
PI=3.141593
Q=1-P:T=SQR(-2*LOG(Q))
A0=2.30753:a1=.27061
B1=.99229:B2=.0481
NU=A0+a1*T
DE=1+B1*T+B2*T*T
X=T-NU/DE

'CUMULATIVE NORMAL
L0: Z=1/SQR(2*PI)*EXP(-X*X/2)
IF X>2 GOTO L1
V=25-13*X*X
FOR N=11 TO 0 STEP-1
U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V
V=U:NEXT N
F=.5-Z*X/V
W=Q-F
GOTO L2
L1:V=X+30
FOR N=29 TO 1 STEP-1
U=X+N/V
V=U:NEXT N
F=Z/V:W=Q-F:GOTO L2

L2:L=L+1
R=X:X=X-W/Z
E=ABS(R-X)
IF E>.001 GOTO L0
PRINT "SAFETY INDEX IS"
PRINT USING "##.####";X
GOTO LL
END

'RELIABILITY FROM  SAFETY INDEX

'NORMIN (0.5,P,1)
DEFDBL A-Z
'INPUT"P=";P:PI=3.141593
PI=3.141593
'Q=1-P:T=SQR(-2*LOG(Q))
'A0=2.30753:A1=.27061
'B1=.99229:B2=.0481
'NU=A0+a1*T
'DE=1+B1*T+B2*T*T
'X=T-NU/DE

'CUMULATIVE NORMAL

INPUT"X=";X
Z=1/SQR(2*PI)*EXP(-X*X/2)
IF X>2 GOTO L1
V=25-13*X*X
FOR N=11 TO 0 STEP-1
U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V
```

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13. ABSTRACT (Maximum 200 words)

The cause of the long-suspected excessive conservatism in the prevailing structural deterministic safety factor has been identified as an inherent violation of the error propagation laws when reducing statistical data to deterministic values and then combining them algebraically through successive structural computational processes. These errors are restricted to the applied stress computations, and because mean and variations of the tolerance limit format are added, the errors are positive, serially cumulative, and excessively conservative. Reliability methods circumvent these errors and provide more efficient and uniform safe structures. The document is a tutorial on the deficiencies and nature of the current safety factor and of its improvement and transition to absolute reliability.

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```
V=U:NEXT N
F=.5-Z*X/V:F=1-F
GOTO L2
L1:V=X+30
FOR N=29 TO 1 STEP-1
U=X+N/V
V=U:NEXT N
F=Z/V:F=1-F
L2:
PRINT F
END
```

D. Mises Criterion Program

```
'ERROR PROPAGATION METHOD;
' MISES CRITERION
DEFDBL A-Z
INPUT"NUMBER OF NORMAL STRESS=",NS
DIM STATIC NSM(3),NSSD(3),NSNF(3),
NSFD(3),LNS(3)

FOR I=1 TO NS
PRINT "NORMAL LOAD MEAN(";I;")="
INPUT NSM(I)
PRINT"NORMAL LOAD STD. DEVIATION(";I;")="
INPUT  NSSD(I)
PRINT"NORMAL LOAD N-FACTOR(";I;")="
INPUT NSNF(I)
NEXT I

INPUT "NUMBER OF SHEAR STRESSES=",MS
DIM STATIC SSM(3),SSSD(3),SSNF(3),SSFD(3),
LSS(3)
FOR I=1 TO MS
PRINT "SHEAR LOAD MEAN(";I;")="
INPUT SSM(I)
PRINT "SHEAR LOAD STD. DEVIATION(";I;")="
INPUT SSSD(I)
PRINT "SHEAR LOAD N-FACTOR(";I;")="
INPUT SSNF(I)
NEXT I

'CALCULATION OF SYSTEM MEAN
S1=0:FOR I=1 TO NS:S1=S1+NSM(I)^2:NEXT I
S2=0:FOR I=1 TO MS:S2=S2+SSM(I)^2:NEXT I
MZ1= S1-NSM(1)*NSM(2)-NSM(1)*NSM(3)
MZ2=MZ1-NSM(2)*NSM(3)+3*S2
MZ= SQR(MZ2)
'CALCULATION OF DERIVATIVES
```

```
NSFD(1)=(2*NSM(1)-NSM(2)-NSM(3))/2/MZ
NSFD(2)=(2*NSM(2)-NSM(1)-NSM(3))/2/MZ
NSFD(3)=(2*NSM(3)-NSM(1)-NSM(2))/2/MZ
FOR I=1 TO MS:SSFD(I)=3*SSM(I)/MZ:NEXT I
'CALCULATION OF SUM OF SQUARES OF
' NORMAL STRESSES
S3=0:S4=0:FOR I=1 TO NS
S3=S3+(NSFD(I)*NSSD(I))^2
S4=S4+(NSNF(I)*NSFD(I)*NSSD(I))^2
NEXT I

'CALCULATION OF SUM OF SQUARES OF
' SHEAR STRESSES
S5=0:S6=0: FOR I=1 TO MS
S5=S5+(SSFD(I)*SSSD(I))^2
S6=S6+(SSNF(I)*SSFD(I)*SSSD(I))^2
NEXT I

'CALCULATION OF SYSTEM STANDARD
'AND EFFECTIVE DEVIATIONS
SZ=SQR(S3+S5):SN=SQR(S4+S6)
NE=SN/SZ

'CALCULATION OF SYSTEM COEFFICIENT
' OF VARIATION
ETA=SZ/MZ

'CALCULATION OF SYSTEM TOLERANCE LIMIT
TL=MZ+(NE*SZ)

'CALCULATION OF MISES FUNCTION
FOR I=1 TO NS
LNS(I)=(NSM(I)+NSNF(I)*NSSD(I))^2
NEXT I
FOR I=1 TO MS
LSS(I)=(SSM(I)+SSNF(I)*SSSD(I))^2
NEXT I
FM1=0:FOR I=1 TO NS
FM1=FM1+LNS(I):NEXT I
FM2=0:FOR I=1 TO MS
FM2=FM2+LSS(I):NEXT I
FM= SQR(FM1+3*FM2)

PRINT "COMBINED APPLIED STRESSS =";FM
PRINT "MEAN =";MZ
PRINT "STANDARD DEVIATION ="SZ
PRINT "EFFECTIVE N =";NE
PRINT "COEFFICIENT OF VARIATION =";ETA
PRINT "TOLERANCE LIMIT =";TL
```